VAR and SVAR III: Some equations

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Abstract

This script reviews some basic equations concerning VAR and SVAR estimation with intercept and time trend. VAR(2) $\mathbf{y}_t = \mathbf{c} + \mathbf{a}t + \Theta_1 \mathbf{y}_{t-1} + \Theta_2 \mathbf{y}_{t-2} + \mathbf{e}_t$ used as illustration.

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1 Constant, mean, and impulse-response function

- ▷ No time trend, $\mathbf{a}t = 0$
- ▷ VAR(2) as VMA(∞)

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{c} + \Theta_1 \mathbf{y}_{t-1} + \Theta_2 \mathbf{y}_{t-2} + \mathbf{e}_t \\
\mathbf{y}_t &= \left(\mathbf{I} - \Theta_1 L - \Theta_2 L^2\right)^{-1} \mathbf{c} + \left(\mathbf{I} - \Theta_1 L - \Theta_2 L^2\right)^{-1} \mathbf{e}_t \\
\mathbf{y}_t &= \left(\mathbf{I} - \Theta_1 - \Theta_2\right)^{-1} \mathbf{c} + \left(\mathbf{I} - \Theta_1 L - \Theta_2 L^2\right)^{-1} \mathbf{e}_t
\end{aligned}$$

- ▷ Expected value from VAR(2)

$$\begin{aligned}
E\left[\mathbf{y}_t\right] &= \boldsymbol{\mu} = \mathbf{c} + \Theta_1 E\left[\mathbf{y}_{t-1}\right] + \Theta_2 E\left[\mathbf{y}_{t-2}\right] + E\left[\mathbf{e}_t\right] \\
\boldsymbol{\mu} &= \mathbf{c} + \Theta_1 \boldsymbol{\mu} + \Theta_2 \boldsymbol{\mu} \\
\boldsymbol{\mu} &= \left(\mathbf{I} - \boldsymbol{\Theta_1} - \boldsymbol{\Theta_2}\right)^{-1} \mathbf{c}
\end{aligned}$$

or expected value from VMA(∞)

$$\begin{aligned}
E\left[\mathbf{y}_t\right] &= \boldsymbol{\mu} = \left(\mathbf{I} - \Theta_1 - \Theta_2\right)^{-1} \mathbf{c} + \left(\mathbf{I} - \Theta_1 L - \Theta_2 L^2\right)^{-1} E\left[\mathbf{e}_t\right] \\
\boldsymbol{\mu} &= \left(\mathbf{I} - \boldsymbol{\Theta_1} - \boldsymbol{\Theta_2}\right)^{-1} \mathbf{c}
\end{aligned}$$

or expected value from transformation of VAR(2) in deviation form

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{c} + \Theta_1 \mathbf{y}_{t-1} + \Theta_2 \mathbf{y}_{t-2} + \mathbf{e}_t \\
\mathbf{y}_t - \boldsymbol{\mu} + \boldsymbol{\mu} &= \mathbf{c} + \Theta_1 \mathbf{y}_{t-1} + \Theta_2 \mathbf{y}_{t-2} + (\Theta_1 + \Theta_2)(\boldsymbol{\mu} - \boldsymbol{\mu}) + \mathbf{e}_t \\
(\mathbf{y}_t - \boldsymbol{\mu}) &= (\Theta_1 \boldsymbol{\mu} + \Theta_2 \boldsymbol{\mu} - \boldsymbol{\mu} + \mathbf{c}) + \Theta_1 (\mathbf{y}_{t-1} - \boldsymbol{\mu}) + \Theta_2 (\mathbf{y}_{t-2} - \boldsymbol{\mu}) + \mathbf{e}_t \\
(\mathbf{y}_t - \boldsymbol{\mu}) &= \Theta_1 (\mathbf{y}_{t-1} - \boldsymbol{\mu}) + \Theta_2 (\mathbf{y}_{t-2} - \boldsymbol{\mu}) + \mathbf{e}_t
\end{aligned}$$

confirming

$$\boldsymbol{\mu} = (\mathbf{I} - \boldsymbol{\Theta_1} - \boldsymbol{\Theta_2})^{-1} \mathbf{c}$$

▷ Deviation written as VMA(∞)

$$\begin{aligned}
(\mathbf{y}_t - \boldsymbol{\mu}) &= \Theta_1 (\mathbf{y}_{t-1} - \boldsymbol{\mu}) + \Theta_2 (\mathbf{y}_{t-2} - \boldsymbol{\mu}) + \mathbf{e}_t \\
(\mathbf{y}_t - \boldsymbol{\mu}) &= (\mathbf{I} - \boldsymbol{\Theta_1} - \boldsymbol{\Theta_2})^{-1} \mathbf{e}_t
\end{aligned}$$

▷ Constant term in a VMA(∞) representation is the mean; constant term in VAR expression does not represent the mean; both intercepts (constant or mean) have no influence on the IRF given by the infinite polynomial; the inclusion of an intercept (constant or mean) in the estimation will however influence the AR coefficients which in turn influence the IRF elements

2 Time trend and impulse-response function

▷ VAR(2) as VMA(∞)

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{c} + \mathbf{a}t + \Theta_1 \mathbf{y}_{t-1} + \Theta_2 \mathbf{y}_{t-2} + \mathbf{e}_t \\
\mathbf{y}_t &= \left(\mathbf{I} - \Theta_1 L - \Theta_2 L^2\right)^{-1} (\mathbf{c} + \mathbf{a}t + \mathbf{e}_t) \\
\mathbf{y}_t &= (\mathbf{I} - \Theta_1 - \Theta_2)^{-1} \mathbf{c} + (\mathbf{I} - \Theta_1 - \Theta_2)^{-1} \mathbf{a}t + \left(\mathbf{I} - \Theta_1 L - \Theta_2 L^2\right)^{-1} \mathbf{e}_t
\end{aligned}$$

▷ Transformation of VAR(2) in deviation from trend

$$\begin{aligned}
\mathbf{y}_t &= \mathbf{c} + \mathbf{a}t + \Theta_1 \mathbf{y}_{t-1} + \Theta_2 \mathbf{y}_{t-2} + \mathbf{e}_t \\
\mathbf{y}_t - \mathbf{k}t + \mathbf{k}t &= \mathbf{c} + \mathbf{a}t + \Theta_1 \mathbf{y}_{t-1} + \Theta_2 \mathbf{y}_{t-2} + \mathbf{e}_t \\
&\quad -\Theta_1 \mathbf{k}(t-1) + \Theta_1 \mathbf{k}(t-1) - \Theta_2 \mathbf{k}(t-2) + \Theta_2 \mathbf{k}(t-2) \\
(\mathbf{y}_t - \mathbf{k}t) &= (\mathbf{c} - \boldsymbol{\Theta_1}\mathbf{k} - 2\boldsymbol{\Theta_2}\mathbf{k}) + (\mathbf{a} + \Theta_1 \mathbf{k} + \Theta_2 \mathbf{k} - \mathbf{k})\, t \\
&\quad + \Theta_1 (\mathbf{y}_{t-1} - \mathbf{k}(t-1)) + \Theta_2 (\mathbf{y}_{t-2} - \mathbf{k}(t-2)) + \mathbf{e}_t
\end{aligned}$$

implying

$$\begin{aligned}
\mathbf{0} &= \mathbf{a} + \Theta_1 \mathbf{k} + \Theta_2 \mathbf{k} - \mathbf{k} \\
\mathbf{a} &= (\mathbf{I} - \boldsymbol{\Theta_1} - \Theta_2)\, \mathbf{k} \\
\mathbf{k} &= (\mathbf{I} - \boldsymbol{\Theta_1} - \boldsymbol{\Theta_2})^{-1} \mathbf{a}
\end{aligned}$$

hence

$$(\mathbf{y}_t - \mathbf{k}t) = \mathbf{c} - \boldsymbol{\Theta_1}\mathbf{k} - 2\boldsymbol{\Theta_2}\mathbf{k} + \Theta_1 (\mathbf{y}_{t-1} - \mathbf{k}(t-1)) + \Theta_2 (\mathbf{y}_{t-2} - \mathbf{k}(t-2)) + \mathbf{e}_t$$

▷ Expectation

$$\begin{aligned}
E\left[\mathbf{y}_t - \mathbf{k}t\right] &= \boldsymbol{\mu} = \mathbf{c} - \Theta_{\mathbf{1}}\mathbf{k} - 2\Theta_{\mathbf{2}}\mathbf{k} \\
&\quad + \Theta_1 E\left[\mathbf{y}_{t-1} - \mathbf{k}\left(t-1\right)\right] + \Theta_2 E\left[\mathbf{y}_{t-2} - \mathbf{k}\left(t-2\right)\right] + E\left[\mathbf{e}_t\right] \\
\boldsymbol{\mu} &= \mathbf{c} - \Theta_{\mathbf{1}}\mathbf{k} - 2\Theta_{\mathbf{2}}\mathbf{k} + \Theta_1\boldsymbol{\mu} + \Theta_2\boldsymbol{\mu} \\
\boldsymbol{\mu} &= \left(\mathbf{I} - \Theta_{\mathbf{1}} - \Theta_{\mathbf{2}}\right)^{-1}\left(\mathbf{c} - \Theta_{\mathbf{1}}\mathbf{k} - 2\Theta_{\mathbf{2}}\mathbf{k}\right)
\end{aligned}$$

▷ Comment similar to the one concerning the inclusion of an intercept

References

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