# RATIONAL EXPECTATIONS MODELS

Nicolas A. CUCHE

April 11, 1999

**Abstract**

A few rational expectations models and their solutions. All in discrete time.

# 1 Cagan Model, Deterministic

## 1.1 Equations

$$\begin{aligned} \text{Fisher Equation} &: \quad r_t = r^* + E_t(p_{t+1} - p_t) \\ \text{Money Demand} &: \quad m_t - p_t = -\gamma r_t \\ \gamma &> 0, \text{ to solve for } p_t \end{aligned}$$

## 1.2 Solution

### 1.2.1 Method of Iterated Expectations

3 steps:

1. RF for the interest variable.
2. Expectation of the RF for $t + i$.
3. Substitution between 1. and 2.

$\boxed{1.}$ Reduced form for $p_t$:

$$\begin{aligned} m_t - p_t &= -\gamma \left( r^* + E_t(p_{t+1} - p_t) \right) \\ -p_t &= -\gamma \left( r^* + E_t p_{t+1} - p_t \right) - m_t \end{aligned}$$

$$\begin{aligned} p_t + \gamma p_t &= \gamma r + \gamma E_t p_{t+1} + m_t \\ p_t &= \left(\frac{\gamma}{1+\gamma}\right) r^* + \left(\frac{\gamma}{1+\gamma}\right) E_t p_{t+1} + \left(\frac{1}{1+\gamma}\right) m_t \end{aligned}$$

2. Expectation of this equation for $E_t p_{t+i}$:

$$\begin{aligned} E_t p_{t+i} &= \left(\frac{\gamma}{1+\gamma}\right) r^* + \left(\frac{\gamma}{1+\gamma}\right) E_t E_{t+i} p_{t+i+1} + \left(\frac{1}{1+\gamma}\right) E_t m_{t+i} \\ E_t p_{t+i} &= \left(\frac{\gamma}{1+\gamma}\right) r^* + \left(\frac{\gamma}{1+\gamma}\right) E_t p_{t+i+1} + \left(\frac{1}{1+\gamma}\right) E_t m_{t+i} \end{aligned}$$

and by analogy:

$$\begin{aligned} E_t p_{t+i+1} &= \left(\frac{\gamma}{1+\gamma}\right) r^* + \left(\frac{\gamma}{1+\gamma}\right) E_t p_{t+i+2} + \left(\frac{1}{1+\gamma}\right) E_t m_{t+i+1} \\ E_t p_{t+i+2} &= \left(\frac{\gamma}{1+\gamma}\right) r^* + \left(\frac{\gamma}{1+\gamma}\right) E_t p_{t+i+3} + \left(\frac{1}{1+\gamma}\right) E_t m_{t+i+2} \\ &\vdots \end{aligned}$$

solve by substitution:

$$\begin{aligned} E_t p_{t+i} &= \left(\frac{\gamma}{1+\gamma}\right) r^* + \left(\frac{1}{1+\gamma}\right) E_t m_{t+i} + \left(\frac{\gamma}{1+\gamma}\right) \cdot \\ &\quad \left[\left(\frac{\gamma}{1+\gamma}\right) r^* + \left(\frac{\gamma}{1+\gamma}\right) E_t p_{t+i+2} + \left(\frac{1}{1+\gamma}\right) E_t m_{t+i+1}\right] \\ E_t p_{t+i} &= \left[\left(\frac{\gamma}{1+\gamma}\right) + \left(\frac{\gamma}{1+\gamma}\right)^2\right] r^* + \left(\frac{\gamma}{1+\gamma}\right)^2 E_t p_{t+i+2} \\ &\quad + \left(\frac{1}{1+\gamma}\right)\left[E_t m_{t+i} + \left(\frac{\gamma}{1+\gamma}\right) E_t m_{t+i+1}\right] \\ E_t p_{t+i} &= \left[\left(\frac{\gamma}{1+\gamma}\right) + \left(\frac{\gamma}{1+\gamma}\right)^2 + \left(\frac{\gamma}{1+\gamma}\right)^3\right] r^* + \left(\frac{1}{1+\gamma}\right) \cdot \end{aligned}$$

$$
\begin{aligned}
& \left[E_t m_{t+i} + \left(\frac{\gamma}{1+\gamma}\right) E_t m_{t+i+1} + \left(\frac{\gamma}{1+\gamma}\right)^2 E_t m_{t+i+2}\right] \\
& + \left(\frac{\gamma}{1+\gamma}\right)^3 E_t p_{t+i+3} \\
& \vdots \\
E_t p_{t+i} &= \gamma r^* + \left(\frac{1}{1+\gamma}\right) \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma}\right)^j E_t m_{t+i+j}
\end{aligned}
$$

3. Plug $E_t p_{t+1}$ in $p_t$:

$$
\begin{aligned}
p_t &= \left(\frac{\gamma}{1+\gamma}\right) r^* + \left(\frac{\gamma}{1+\gamma}\right) E_t p_{t+1} + \left(\frac{1}{1+\gamma}\right) m_t \\
p_t &= \left(\frac{\gamma}{1+\gamma}\right) r^* + \left(\frac{\gamma}{1+\gamma}\right) \left[\gamma r^* + \left(\frac{1}{1+\gamma}\right) \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma}\right)^j E_t m_{t+1+j}\right] \\
& + \left(\frac{1}{1+\gamma}\right) m_t \\
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right) \sum_{j=0}^{\infty} \left(\frac{\gamma}{1+\gamma}\right)^{j+1} E_t m_{t+1+j} + \left(\frac{1}{1+\gamma}\right) m_t \\
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right) \left[m_t + \sum_{j=1}^{\infty} \left(\frac{\gamma}{1+\gamma}\right)^j E_t m_{t+j}\right]
\end{aligned}
$$

### 1.2.2 Method of U.C.

5 steps:

1. RF for the interest variable.
2. Hypothesis.
3. Substitution.
4. Elimination.
5. Substitution.

1. Reduced form for $p_t$:

$$
p_t = \left(\frac{\gamma}{1+\gamma}\right) r^* + \left(\frac{\gamma}{1+\gamma}\right) E_t p_{t+1} + \left(\frac{1}{1+\gamma}\right) m_t
$$

$\boxed{2.}$ Hypothesis for the solution of $p_t$ with U.C.:

$$p_t = \rho + \sum_{i=0}^{\infty} \mu_i m_{t-i} + \sum_{j=1}^{\infty} \lambda_j E_t m_{t+j}$$

the same hypothesis for $p_{t+1}$:

$$p_{t+1} = \rho + \sum_{i=0}^{\infty} \mu_i m_{t-i+1} + \sum_{j=1}^{\infty} \lambda_j E_{t+1} m_{t+1+j}$$

and its expected expression:

$$\begin{aligned} E_t p_{t+1} &= \rho + \sum_{i=0}^{\infty} \mu_i E_t m_{t-i+1} + \sum_{j=1}^{\infty} \lambda_j E_t E_{t+1} m_{t+1+j} \\ &= \rho + \mu_0 E_t m_{t+1} + \sum_{i=1}^{\infty} \mu_i m_{t-i+1} + \sum_{j=1}^{\infty} \lambda_j E_t m_{t+1+j} \end{aligned}$$

$\boxed{3.}$ Plug $p_t$ and $E_t p_{t+1}$ in the reduced form:

$$\begin{aligned} \sum_{i=0}^{\infty} \mu_i m_{t-i} + \sum_{j=1}^{\infty} \lambda_j E_t m_{t+j} &= \left(\frac{\gamma}{1+\gamma}\right) r^* + \left(\frac{1}{1+\gamma}\right) m_t - \rho \\ &+ \left(\frac{\gamma}{1+\gamma}\right) \left[ \begin{array}{c} \rho + \mu_0 E_t m_{t+1} \\ + \sum_{i=1}^{\infty} \mu_i m_{t+1-i} \\ + \sum_{j=1}^{\infty} \lambda_j E_t m_{t+1+j} \end{array} \right] \end{aligned}$$

$\boxed{4.}$ Everything equals zero except the $m_t$ multiplicators:

$$\begin{aligned} \mu_0 m_t &= \left(\frac{1}{1+\gamma}\right) m_t + \left(\frac{\gamma}{1+\gamma}\right) \mu_1 m_t \\ \mu_0 &= \left(\frac{1}{1+\gamma}\right) + \left(\frac{\gamma}{1+\gamma}\right) \mu_1 \end{aligned}$$

the same for $m_{t-i}$:

$$
\begin{aligned}
\mu_i m_{t-i} &= \left(\frac{\gamma}{1+\gamma}\right)\mu_{i+1}m_{t-i} \\
\mu_i &= \left(\frac{\gamma}{1+\gamma}\right)\mu_{i+1} \\
\mu_{i-1} &= \left(\frac{\gamma}{1+\gamma}\right)\mu_i \\
\mu_i &= \left(\frac{1+\gamma}{\gamma}\right)\mu_{i-1} \\
\mu_i &= \left(\frac{1+\gamma}{\gamma}\right)\left(\frac{1+\gamma}{\gamma}\right)\mu_{i-2} \\
&\vdots \\
\mu_i &= \left(\frac{1+\gamma}{\gamma}\right)^{i-1}\mu_1
\end{aligned}
$$

the same for $E_t m_{t+j}$:

$$
\begin{aligned}
\lambda_j E_t m_{t+j} &= \left(\frac{\gamma}{1+\gamma}\right)\lambda_{j-1}E_t m_{t+j} \\
\lambda_j &= \left(\frac{\gamma}{1+\gamma}\right)\lambda_{j-1} \\
\lambda_j &= \left(\frac{\gamma}{1+\gamma}\right)\left(\frac{\gamma}{1+\gamma}\right)\lambda_{j-2} \\
&\vdots \\
\lambda_j &= \left(\frac{\gamma}{1+\gamma}\right)^{j-1}\lambda_1
\end{aligned}
$$

the same for $cst$:

$$
\begin{aligned}
0 &= \left(\frac{\gamma}{1+\gamma}\right)r^* - \rho + \left(\frac{\gamma}{1+\gamma}\right)\rho \\
0 &= \gamma r^* - (1+\gamma)\rho + \gamma\rho
\end{aligned}
$$

$$\begin{aligned}
0 &= \gamma r^* - \rho - \gamma\rho + \gamma\rho \\
0 &= \gamma r^* - \rho \\
\rho &= \gamma r^*
\end{aligned}$$

$\boxed{5.}$ Rewrite the assumption with the new results:

$$\begin{aligned}
p_t &= \rho + \sum_{i=0}^{\infty} \mu_i m_{t-i} + \sum_{j=1}^{\infty} \lambda_j E_t m_{t+j} \\
p_t &= \rho + \mu_0 m_t + \sum_{i=1}^{\infty} \mu_i m_{t-i} + \sum_{j=1}^{\infty} \lambda_j E_t m_{t+j} \\
p_t &= \gamma r^* + \left[ \left( \frac{1}{1+\gamma} \right) + \left( \frac{\gamma}{1+\gamma} \right) \mu_1 \right] m_t \\
&\quad + \sum_{i=1}^{\infty} \left( \frac{1+\gamma}{\gamma} \right)^{i-1} \mu_1 m_{t-i} \\
&\quad + \sum_{j=1}^{\infty} \left( \frac{\gamma}{1+\gamma} \right)^{j-1} \lambda_1 E_t m_{t+j}
\end{aligned}$$

To remove the explosive behaviour, assume that $\mu_1 = 0$, because $\left(\frac{1+\gamma}{\gamma}\right)^{i-1}$ exploses. So, $\mu_0 = \left(\frac{1}{1+\gamma}\right)$. And then, calculate $\lambda_1$; if everything equals zero, except the $E_t m_{t+1}$ multiplicators , then, $\lambda_1 E_t m_{t+1} = \left(\frac{\gamma}{1+\gamma}\right) \mu_0 E_t m_{t+1}$. $\lambda_1 = \left(\frac{\gamma}{1+\gamma}\right) \mu_0$ so $\lambda_1 = \frac{\gamma}{(1+\gamma)^2}$.

$$\begin{aligned}
p_t &= \gamma r^* + \left( \frac{1}{1+\gamma} \right) m_t + \sum_{j=1}^{\infty} \left( \frac{\gamma}{1+\gamma} \right)^{j-1} \frac{\gamma}{(1+\gamma)^2} E_t m_{t+j} \\
p_t &= \gamma r^* + \left( \frac{1}{1+\gamma} \right) m_t + \left( \frac{1}{1+\gamma} \right) \sum_{j=1}^{\infty} \left( \frac{\gamma}{1+\gamma} \right)^{j} E_t m_{t+j} \\
p_t &= \gamma r^* + \left( \frac{1}{1+\gamma} \right) \left[ m_t + \sum_{j=1}^{\infty} \left( \frac{\gamma}{1+\gamma} \right)^{j} E_t m_{t+j} \right]
\end{aligned}$$

## 1.3 Simulations

- $m_t = \bar{m}, \forall t$. The price level, the nominal interest rate, and the real balance in SS are then:

$$
\begin{aligned}
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right)\left[\bar{m} + \bar{m}\sum_{j=1}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^j\right] \\
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right)\left[\bar{m}\left(1 + \sum_{j=1}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^j\right)\right] \\
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right)\left[\bar{m}\left(1+\gamma\right)\right] \\
p_t &= \gamma r^* + \bar{m}
\end{aligned}
$$

$$
\begin{aligned}
r_t &= r^* + E_t(p_{t+1} - p_t) \\
r_t &= r^*
\end{aligned}
$$

$$
m_t - p_t = \bar{m} - \gamma r - \bar{m} = -\gamma r
$$

- $m_t = m_0 + \mu t$. The price level, the nominal interest rate, and the real balance in SS are then:

$$
\begin{aligned}
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right)\left[m_t + \sum_{j=1}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^j E_t m_{t+j}\right] \\
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right)\left[m_0 + \mu t + \sum_{j=1}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^j \left(m_0 + \mu\left(t+j\right)\right)\right] \\
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right)\begin{bmatrix} m_0 + m_0\sum_{j=1}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^j \\ +\mu t + \sum_{j=1}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^j \mu t \\ +\sum_{j=1}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^j \mu j \end{bmatrix} \\
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right)\begin{bmatrix} m_0\left(1+\gamma\right) \\ +\mu t\left(1+\gamma\right) \\ +\mu\sum_{j=1}^{\infty} j\left(\frac{\gamma}{1+\gamma}\right)^j \end{bmatrix}
\end{aligned}
$$

$$\begin{aligned}
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right)\left[m_0(1+\gamma) + \mu t(1+\gamma) + \mu(1+\gamma)\gamma\right] \\
p_t &= \gamma r^* + m_0 + \mu t + \mu\gamma \\
p_t &= \gamma(r^* + \mu) + m_0 + \mu t \\
p_t &= m_t + \gamma(r^* + \mu)
\end{aligned}$$

$$\begin{aligned}
r_t &= r^* + E_t(p_{t+1} - p_t) \\
r_t &= r^* + \left[(\gamma(r^*+\mu) + m_0 + \mu(t+1)) - (\gamma(r^*+\mu) + m_0 + \mu t)\right] \\
r_t &= r^* + \mu(t+1) - \mu t \\
r_t &= r^* + \mu
\end{aligned}$$

$$\begin{aligned}
m_t - p_t &= m_0 + \mu t - \gamma(r^*+\mu) - m_0 - \mu t \\
m_t - p_t &= -\gamma(r^*+\mu)
\end{aligned}$$

- $m_t = m_0(1+\mu)^t$. The price level, the nominal interest rate, and the real balance in SS are then:

$$\begin{aligned}
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right)\left[m_t + \sum_{j=1}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^j E_t m_{t+j}\right] \\
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right)\left[\begin{array}{c} m_0(1+\mu)^t \\ +\sum_{j=1}^{\infty}\left(\frac{\gamma}{1+\gamma}\right)^j m_0(1+\mu)^{t+j} \end{array}\right] \\
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right)\left[\begin{array}{c} m_0(1+\mu)^t \\ +\sum_{j=1}^{\infty} m_0\left(\frac{\gamma(1+\mu)}{(1+\gamma)}\right)^j (1+\mu)^t \end{array}\right] \\
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right) m_0(1+\mu)^t\left[1 + \sum_{j=1}^{\infty}\left(\frac{\gamma(1+\mu)}{(1+\gamma)}\right)^j\right] \\
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right) m_0(1+\mu)^t\left[\frac{1}{1 - \frac{\gamma(1+\mu)}{(1+\gamma)}}\right]
\end{aligned}$$

$$\begin{aligned}
p_t &= \gamma r^* + \left(\frac{1}{1+\gamma}\right) m_0 \left(1+\mu\right)^t \left[\frac{(1+\gamma)}{1-\gamma\mu}\right] \\
p_t &= \gamma r^* + m_0 \left(1+\mu\right)^t \left(\frac{1}{1-\gamma\mu}\right) \\
p_t &= \gamma r^* + m_t \left(\frac{1}{1-\gamma\mu}\right)
\end{aligned}$$

$$\begin{aligned}
p_{t+1} - p_t &= \gamma r^* + m_{t+1} \left(\frac{1}{1-\gamma\mu}\right) - \gamma r^* - m_t \left(\frac{1}{1-\gamma\mu}\right) \\
p_{t+1} - p_t &= \left(\frac{1}{1-\gamma\mu}\right) (m_{t+1} - m_t) \\
p_{t+1} - p_t &= m_t \left(\frac{\mu}{1-\gamma\mu}\right)
\end{aligned}$$

$$\begin{aligned}
r_t &= r^* + E_t(p_{t+1} - p_t) \\
&= r^* + m_t \left(\frac{\mu}{1-\gamma\mu}\right)
\end{aligned}$$

$$\begin{aligned}
m_t - p_t &= m_0 \left(1+\mu\right)^t - \gamma r^* - m_0 \left(1+\mu\right)^t \left(\frac{1}{1-\gamma\mu}\right) \\
&= m_0 \left(1+\mu\right)^t \left[1 - \left(\frac{1}{1-\gamma\mu}\right)\right] - \gamma r^* \\
&= m_0 \left(1+\mu\right)^t \left[\frac{-\gamma\mu}{1-\gamma\mu}\right] - \gamma r^* \\
&= m_t \left(\frac{\gamma\mu}{\gamma\mu - 1}\right) - \gamma r^*
\end{aligned}$$

# 2 Method of U.C. Money Demand and Supply, Stochastic

## 2.1 Equations

$$\text{Money Demand} \quad : \quad m_t - p_t = \gamma + \alpha \left(E_t p_{t+1} - p_t\right) + u_t$$

$$\begin{aligned}
\text{Money Supply} &: \quad m_t = \mu_0 + \mu_1 m_{t-1} + e_t \\
\alpha &< 0 \\
|\mu_1| &< 1, \text{ to solve for } p_t
\end{aligned}$$

## 2.2 Solution U.C.

1. Reduced form:

$$\begin{aligned}
-p_t &= \gamma + \alpha \left( E_t p_{t+1} - p_t \right) + u_t - \left( \mu_0 + \mu_1 m_{t-1} + e_t \right) \\
p_t &= -\gamma - \alpha E_t p_{t+1} + \alpha p_t - u_t + \mu_0 + \mu_1 m_{t-1} + e_t \\
p_t &= \left( \frac{1}{1-\alpha} \right) (\mu_0 - \gamma) - \left( \frac{1}{1-\alpha} \right) \alpha E_t p_{t+1} \\
&\quad + \left( \frac{1}{1-\alpha} \right) \mu_1 m_{t-1} + \left( \frac{1}{1-\alpha} \right) (e_t - u_t)
\end{aligned}$$

2. Hypothesis for the solution of $p_t$:

$$p_t = \pi_0 + \pi_1 m_{t-1} + \pi_2 e_t + \pi_3 u_t$$

$$\begin{aligned}
p_{t+1} &= \pi_0 + \pi_1 m_t + \pi_2 e_{t+1} + \pi_3 u_{t+1} \\
E_t p_{t+1} &= \pi_0 + \pi_1 m_t \\
E_t p_{t+1} &= \pi_0 + \pi_1 \left( \mu_0 + \mu_1 m_{t-1} + e_t \right)
\end{aligned}$$

3. Substitution

$$\begin{aligned}
p_t &= \left( \frac{1}{1-\alpha} \right) (\mu_0 - \gamma) - \left( \frac{1}{1-\alpha} \right) \alpha E_t p_{t+1} \\
&\quad + \left( \frac{1}{1-\alpha} \right) \mu_1 m_{t-1} + \left( \frac{1}{1-\alpha} \right) (e_t - u_t) \\
\pi_0 &= \left( \frac{1}{1-\alpha} \right) (\mu_0 - \gamma) - \left( \frac{\alpha}{1-\alpha} \right) \left( \pi_0 + \pi_1 \mu_0 + \pi_1 \mu_1 m_{t-1} + \pi_1 e_t \right) \\
&\quad + \left( \frac{1}{1-\alpha} \right) \mu_1 m_{t-1} + \left( \frac{1}{1-\alpha} \right) (e_t - u_t) \\
&\quad - \pi_1 m_{t-1} - \pi_2 e_t - \pi_3 u_t
\end{aligned}$$

$\boxed{4.}$ Everything equals zero except the *cst* multiplicators:

$$\begin{aligned}
\pi_0 &= \left(\frac{1}{1-\alpha}\right)(\mu_0-\gamma)-\left(\frac{\alpha\left(\pi_0+\pi_1\mu_0\right)}{1-\alpha}\right)\\
(1-\alpha)\,\pi_0 &= \mu_0-\gamma-\alpha\pi_0-\alpha\pi_1\mu_0\\
\pi_0 &= \mu_0\left(1-\alpha\pi_1\right)-\gamma
\end{aligned}$$

the same for $m_{t-1}$:

$$\begin{aligned}
0 &= -\left(\frac{\alpha}{1-\alpha}\right)\pi_1\mu_1 m_{t-1}+\left(\frac{1}{1-\alpha}\right)\mu_1 m_{t-1}-\pi_1 m_{t-1}\\
0 &= \left(\frac{-\alpha}{1-\alpha}\right)\pi_1\mu_1+\left(\frac{1}{1-\alpha}\right)\mu_1-\pi_1\\
0 &= -\alpha\pi_1\mu_1+\mu_1-\pi_1\left(1-\alpha\right)\\
\mu_1 &= \pi_1\left[\alpha\mu_1+(1-\alpha)\right]\\
\pi_1 &= \frac{\mu_1}{1-\alpha+\alpha\mu_1}
\end{aligned}$$

recalculate $\pi_0$:

$$\begin{aligned}
\pi_0 &= \mu_0\left(1-\alpha\pi_1\right)-\gamma\\
\pi_0 &= \mu_0\left(1-\alpha\left(\frac{\mu_1}{1-\alpha+\alpha\mu_1}\right)\right)-\gamma\\
\pi_0 &= \frac{\mu_0\left(1-\alpha\right)}{1-\alpha+\alpha\mu_1}-\gamma
\end{aligned}$$

the same for $e_t$:

$$\begin{aligned}
0 &= -\left(\frac{\alpha}{1-\alpha}\right)\pi_1 e_t+\left(\frac{1}{1-\alpha}\right)e_t-\pi_2 e_t\\
0 &= -\left(\frac{\alpha}{1-\alpha}\right)\pi_1+\left(\frac{1}{1-\alpha}\right)-\pi_2\\
0 &= -\alpha\pi_1+1-\pi_2\left(1-\alpha\right)\\
\pi_2 &= \frac{1-\alpha\pi_1}{(1-\alpha)}=\frac{1-\alpha\left(\frac{\mu_1}{1-\alpha+\alpha\mu_1}\right)}{(1-\alpha)}
\end{aligned}$$

$$\pi_2 = \frac{1}{1-\alpha+\alpha\mu_1}$$

the same for $u_t$:

$$\begin{aligned}
0 &= \left(\frac{1}{1-\alpha}\right)(-u_t) - \pi_3 u_t \\
0 &= \left(\frac{-1}{1-\alpha}\right) - \pi_3 \\
\pi_3 &= \frac{-1}{1-\alpha}
\end{aligned}$$

5. Rewrite the assumption with the new results:

$$\begin{aligned}
p_t &= \pi_0 + \pi_1 m_{t-1} + \pi_2 e_t + \pi_3 u_t \\
p_t &= \left(\frac{\mu_0(1-\alpha)}{1-\alpha+\alpha\mu_1} - \gamma\right) + \left(\frac{\mu_1}{1-\alpha+\alpha\mu_1}\right) m_{t-1} \\
&\quad + \left(\frac{1}{1-\alpha+\alpha\mu_1}\right) e_t + \left(\frac{-1}{1-\alpha}\right) u_t \\
p_t &= \left(\frac{\mu_0(1-\alpha)}{1-\alpha+\alpha\mu_1}\right) - \gamma + \left(\frac{\mu_1 m_{t-1}}{1-\alpha+\alpha\mu_1}\right) \\
&\quad + \left(\frac{e_t}{1-\alpha+\alpha\mu_1}\right) - \left(\frac{u_t}{1-\alpha}\right) \\
p_t &= \frac{\mu_0(1-\alpha) + \mu_1 m_{t-1} + e_t}{(1-\alpha+\alpha\mu_1)} - \gamma - \left(\frac{u_t}{1-\alpha}\right)
\end{aligned}$$

## 2.3 Simulations

- $e_t = u_t = 0$ so $m_t = \mu_0 + \mu_1 m_{t-1}$.

$$\begin{aligned}
m_t &= \mu_0 + \mu_1 m_{t-1} \\
m_t &= \mu_0 + \mu_1(\mu_0 + \mu_1 m_{t-2}) \\
m_t &= \mu_0 + \mu_1(\mu_0 + \mu_1(\mu_0 + \mu_1 m_{t-3}))
\end{aligned}$$

$$\begin{aligned}
&\vdots \\
m_t &= \mu_0 + \mu_1\mu_0 + \mu_1^2\mu_0 + \mu_1^3 m_{t-3} \\
m_t &= \frac{\mu_0}{1-\mu_1}
\end{aligned}$$

$$\begin{aligned}
p_t &= \frac{\mu_0\left(1-\alpha\right) + \mu_1\left(\frac{\mu_0}{1-\mu_1}\right)}{\left(1-\alpha+\alpha\mu_1\right)} - \gamma \\
p_t &= \frac{\mu_0\left[\frac{(1-\alpha)(1-\mu_1)+\mu_1}{(1-\mu_1)}\right]}{\left(1-\alpha+\alpha\mu_1\right)} - \gamma \\
p_t &= \frac{\mu_0}{\left(1-\mu_1\right)} - \gamma
\end{aligned}$$

- $e_t = u_t = \mu_1 = 0$ so $m_t = \mu_0$.

$$\begin{aligned}
p_t &= \frac{\mu_0\left(1-\alpha\right) + \mu_1 m_{t-1} + e_t}{\left(1-\alpha+\alpha\mu_1\right)} - \gamma - \left(\frac{u_t}{1-\alpha}\right) \\
p_t &= \frac{\mu_0\left(1-\alpha\right) + \mu_1\mu_0}{\left(1-\alpha+\alpha\mu_1\right)} - \gamma \\
p_t &= \mu_0 - \gamma
\end{aligned}$$