International parities

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1 Law Of One Price

1.1 Purchasing Power Parity

▷ **Idea.** Similar goods sell for the same price worldwide. E is the price of foreign currency expressed in domestic unit. P_t is the domestic price level. P_t^* is the foreign price level.

▷ **Equations.** From absolute PPP to approximate relative PPP.

$$\begin{aligned}
P_t &= E_t P_t^* \\
E_t &= \frac{P_t}{P_t^*}, \quad E_{t+1} = \frac{P_{t+1}}{P_{t+1}^*} \\
\frac{E_{t+1}}{E_t} &= \frac{\frac{P_{t+1}}{P_{t+1}^*}}{\frac{P_t}{P_t^*}} \\
\ln\left(\frac{E_{t+1}}{E_t}\right) &= \ln\left(\frac{P_{t+1}}{P_t}\right) - \ln\left(\frac{P_{t+1}^*}{P_t^*}\right) \\
{}_t\hat{E}_{t+1} &\simeq {}_t\hat{P}_{t+1} - {}_t\hat{P}_{t+1}^*
\end{aligned}$$

▷

Summary

APPP	$E_t = \frac{P_t}{P_t^*}$
RPPP	${}_t\hat{E}_{t+1} \simeq {}_t\hat{P}_{t+1} - {}_t\hat{P}_{t+1}^*$

▷ **Difference.** Exact RPPP and approximative RPPP.

$$
\begin{aligned}
\frac{E_{t+1}}{E_t} - 1 &= \frac{\frac{P_{t+1}}{P^*_{t+1}}}{\frac{P_t}{P^*_t}} - 1 \\
{}_t\hat{E}_{t+1} &= \frac{\frac{P_{t+1}}{P_t} - \frac{P^*_{t+1}}{P^*_t} - 1 + 1}{\frac{P^*_{t+1}}{P^*_t}} \\
{}_t\hat{E}_{t+1} &= \frac{{}_t\hat{P}_{t+1} - {}_t\hat{P}^*_{t+1}}{1 + {}_t\hat{P}^*_{t+1}} \\
{}_t\hat{E}_{t+1} + \underbrace{{}_t\hat{E}_{t+1}\ {}_t\hat{P}^*_{t+1}}_{\to 0} &= {}_t\hat{P}_{t+1} - {}_t\hat{P}^*_{t+1}
\end{aligned}
$$

▷ **Interpretation.** Currency of countries with relatively more inflation than their partners depreciates.

1.2 Real Exchange Rate

▷ **Definition.** Cost of foreign goods in term of domestic goods, defined as the nominal exchange rate adjusted by prices at home and abroad.

▷ **Equations.**

$$
\begin{aligned}
\lambda_t &= \frac{P_t^{traded(domestic\ unit)}}{P_t^{nontraded(domestic\ unit)}} \\
\lambda_t &= \frac{EP_t^{*(foreign\ currency)}}{P_t^{nontraded(domestic\ unit)}} = \frac{EP^*_t}{P_t}
\end{aligned}
$$

▷ **Interpretation.** A real exchange rate of $\lambda_t = 1$ corresponds to the APPP $P_t = E_t P^*_t$. A real exchange rate of 1 means that the cost of foreign goods and domestic goods are the same.

2 Profit-Seeking Arbitrage Activity

2.1 Uncovered Interest Rate Parity

▷ **Idea.** E_t is the domestic price of foreign currency. $1/E_t$ is the foreign price of a domestic unit. An investor invests at home 1 domestic unit and earns $(1 + i_t)$. If however he goes abroad and invests $1/E_t$, he will receive $\frac{1}{E_t}(1 + i^*_t)$ in foreign terms. In order to compare, he transforms this amount in domestic value at the end of the holding period and gets $\left(\frac{(1+i^*_t)}{E_t}\right) E_{t+1}$. The two payoffs, domestic and abroad, must be equal in presence of arbitrage opportunities.

▷ **Equation.**

$$\begin{aligned}(1+i_t) &= \frac{E_{t+1}}{E_t}(1+i_t^*) \\ \ln(1+i_t) &= \ln\left(\frac{E_{t+1}}{E_t}\right)+\ln(1+i_t^*) \\ i_t &\tilde{=} {}_t\hat{E}_{t+1}+i_t^* \\ {}_t\hat{E}_{t+1} &\tilde{=} i_t-i_t^*\end{aligned}$$

▷ **Difference**.

$$\begin{aligned}(1+i_t) &= \frac{E_{t+1}}{E_t}(1+i_t^*) \\ \frac{(1+i_t)}{(1+i_t^*)}-1 &= \frac{E_{t+1}}{E_t}-1 \\ \frac{i_t-i_t^*}{1+i_t^*} &= \frac{E_{t+1}-E_t}{E_t} = {}_t\hat{E}_{t+1} \\ {}_t\hat{E}_{t+1}+\underbrace{i_t^*\ {}_t\hat{E}_{t+1}}_{\to 0} &= i_t-i_t^*\end{aligned}$$

▷ **Interpretation**. If the UIRP is violated, profit activities are possible again. For example, one trader notices that $i_t \tilde{=} {}_t\hat{E}_{t+1}+i_t^*$ is not given, e.g. $i_t > {}_t\hat{E}_{t+1}+i_t^*$ meaning that $(1+i_t) > \frac{E_{t+1}}{E_t}(1+i_t^*)$. He then borrows abroad, invests at home, and eventually repays his debt with a marginal benefit. If everybody plays this game, it will induce an increase of the interest rate abroad and a decrease in the domestic interest rate. This will also tend to increase the value of the foreign currency. The parity will be set again. Countries with lower interest rates see their own currency appreciate.

2.2 Covered Interest Rate Parity

▷ **Idea.** If there is a forward exchange rate market, we can use the relative difference between the spot and forward rate instead of the growth rate (E_{t+1} becomes F_t).

▷ **Equations**.

$$\frac{i_t-i_t^*}{1+i_t^*}=\frac{F_t-E_t}{E_t}$$

▷

Summary

UIRP	$\frac{i_t-i_t^*}{1+i_t^*}=\frac{E_{t+1}-E_t}{E_t}={}_t\hat{E}_{t+1}$
UIRP	${}_t\hat{E}_{t+1}\tilde{=}i_t-i_t^*$ or $i_t\tilde{=}i_t^*+{}_t\hat{E}_{t+1}$
CIRP	$\frac{i_t-i_t^*}{1+i_t^*}=\frac{F_t-E_t}{E_t}$

3 Interest Rate and Prices

3.1 Fisher Equation

▷ **Idea.** Real interest rate is the nominal interest rate corrected by the inflation rate. When an investor domestically invests, he gets $(1+i_t)$. But when there is inflation ${}_t\hat{P}_{t+1}$, he observes a decrease in his purchasing power. This is like investing in a 'bad' asset and getting $\frac{1}{1+{}_t\hat{P}_{t+1}}$. So he will receive $\frac{1+i_t}{1+{}_t\hat{P}_{t+1}}$ and that corresponds to a real return of $1+r_t$.

▷ **Equations.**

$$\begin{aligned}
(1+r_t) &= \frac{(1+i_t)}{\left(1+\ {}_t\hat{P}_{t+1}\right)} \\
\ln(1+r_t) &= \ln(1+i_t) - \ln\left(1+\ {}_t\hat{P}_{t+1}\right) \\
r_t &\simeq i_t - {}_t\hat{P}_{t+1}
\end{aligned}$$

▷ **Difference.**

$$\begin{aligned}
(1+r_t) &= \frac{(1+i_t)}{\left(1+\ {}_t\hat{P}_{t+1}\right)} \\
(1+i_t) &= (1+r_t)\left(1+\ {}_t\hat{P}_{t+1}\right) \\
1+i_t &= 1+\ {}_t\hat{P}_{t+1} + r_t + \underbrace{{}_t\hat{P}_{t+1} r_t}_{\to 0}
\end{aligned}$$

3.2 International Fisher Equation

▷ **Idea.** Fisher equation, relative purchasing power parity, and uncovered interest rate parity imply the international Fisher equation: $r_t = r_t^*$. Real interest rates are worldwide equal.

▷ **Equations.**

$$\begin{aligned}
\text{UIRP} &: (1+i_t) = \frac{E_{t+1}}{E_t}(1+i_t^*) \\
\text{APPP} &: \frac{E_{t+1}}{E_t} = \frac{\frac{P_{t+1}}{P_{t+1}^*}}{\frac{P_t}{P_t^*}} \\
\text{FE at home} &: (1+i_t) = (1+r_t)\left(1+\ {}_t\hat{P}_{t+1}\right) \\
\text{FE in USA} &: (1+i_t^*) = (1+r_t^*)\left(1+\ {}_t\hat{P}_{t+1}^*\right)
\end{aligned}$$

$$\Downarrow$$

$$
\begin{aligned}
\frac{(1+i_t)}{(1+i_t^*)} &= \frac{\frac{P_{t+1}}{P_{t+1}^*}}{\frac{P_t}{P_t^*}} \\
\ln\left[\frac{(1+r_t)\left(1+\ {}_t\hat{P}_{t+1}\right)}{(1+r_t^*)\left(1+\ {}_t\hat{P}_{t+1}^*\right)}\right] &= \ln\left[\frac{\frac{P_{t+1}}{P_t}}{\frac{P_{t+1}^*}{P_t^*}}\right] \\
\ln(1+r_t) - \ln(1+r_t^*) &= \ln\left(1+\ {}_t\hat{P}_{t+1}^*\right) - \ln\left(1+\ {}_t\hat{P}_{t+1}\right) \\
&\quad + \ln\left(\frac{P_{t+1}}{P_t}\right) - \ln\left(\frac{P_{t+1}^*}{P_t^*}\right) \\
\ln(1+r_t) - \ln(1+r_t^*) &= 0 \\
r_t &= r_t^*
\end{aligned}
$$

▷

Summary

FE	$i_t \tilde{=} r_t +\ {}_t\hat{P}_{t+1}$
IFE	$r_t = r_t^*$

4 Parities at a Glance

Summary

APPP	$E_t = \frac{P_t}{P_t^*}$
RPPP	${}_t\hat{E}_{t+1} \tilde{=} {}_t\hat{P}_{t+1} -\ {}_t\hat{P}_{t+1}^*$
RER	$\lambda_t = \frac{EP_t^*}{P_t}$
UIRP	${}_t\hat{E}_{t+1} \tilde{=} i_t - i_t^*$
CIRP	$\frac{i_t - i_t^*}{1+i_t^*} = \frac{F_t - E_t}{E_t}$
FE	$i_t \tilde{=} r_t +\ {}_t\hat{P}_{t+1}$
IFE	$r_t = r_t^*$