VAR

- **V**ector **A**uto **R**egression

  Vector → Multivariate
  Auto → Self-explained
  Regression → OLS

1. Definition

Forms, elements, and features

2. What can we do with VAR?

Intuitive part	Estimation
	Dynamics
	Forecasting

3. What is VAR econometrics?

Technical part	VAR(1) as example
	VAR versus SVAR

4. What have we learned with VAR?

Empirical evidence

5. Just do it!

Computer part EViews

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1.Definition

- VAR is an econometric device to model multivariate time series

  or

  VAR is a particular system of multiple regression equations
- Particular
  - All variables of interest are endogenous
  - All equations use same explanatory variables
  - Explanatory variables are mainly lagged variables
- Example: Money and output relation

Money=f(lagged money, lagged output)+shock

Output=f(lagged money, lagged output)+shock

1.Definition
2.What can we do with VAR?
3.What is VAR econometrics (I, II, III, SVAR)?
4.What have we learned with VAR
5. Just do it!

Notes

Notes

2.What can we do with VAR?

Money=f(lagged money, lagged output)+shock
Output=f(lagged money, lagged output)+shock

- I) Estimation

  Coefficients, tests, shocks

  → Intermediate step, No report
  [*ECONOMETRICS* **I]**

- II) Dynamics

  shock the system now, effects on variables over time, impulse-response functions

  → Final step, Plot Response/Time
  [*ECONOMETRICS* **II]**

- III) Forecasting

  Plain forecasting, with current values, forecasting tomorrow's values

  Shock contribution to forecasting errors, variance decomposition

  → Final step, Table MSE/Shocks
  [*ECONOMETRICS* **III]**

3.What is VAR(1) *Econometrics* **I**?

Money=f(lagged money, lagged output)+shock
Output=f(lagged money, lagged output)+shock

- Estimation

$$
\begin{aligned}
m_t &= \omega_{11} m_{t-1} + \omega_{12} x_{t-1} + e_t^m \\
x_t &= \omega_{21} m_{t-1} + \omega_{22} x_{t-1} + e_t^x
\end{aligned}
$$

$$
\begin{aligned}
\mathbf{y}_t &= \begin{pmatrix} m_t \\ x_t \end{pmatrix} \\
\mathbf{y}_{t-1} &= \begin{pmatrix} m_{t-1} \\ x_{t-1} \end{pmatrix}
\end{aligned}
$$

$$\mathbf{y}_t = \Omega \mathbf{y}_{t-1} + \mathbf{e}_t$$

$$\Omega = \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{pmatrix}$$

Notes

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3.What is VAR(1) *Econometrics* **II**?

$$
\begin{aligned}
m_t &= \omega_{11} m_{t-1} + \omega_{12} x_{t-1} + e_t^m \\
x_t &= \omega_{21} m_{t-1} + \omega_{22} x_{t-1} + e_t^x
\end{aligned}
$$

- Dynamics

$$
\begin{aligned}
\Delta e_t^m &\rightarrow \Delta m_{t+l} \\
\Delta e_t^m &\rightarrow \Delta x_{t+l} \\
\Delta e_t^x &\rightarrow \Delta m_{t+l} \\
\Delta e_t^x &\rightarrow \Delta x_{t+l}
\end{aligned}
$$

$$l = 0, \ldots, \infty$$

$$\Delta \mathbf{e}_t \rightarrow \Delta \mathbf{y}_{t+l}$$

3.What is VAR(1) *Econometrics* **II**?

- VAR to VMA

$$
\begin{aligned}
\mathbf{y}_t &= \Omega \mathbf{y}_{t-1} + \mathbf{e}_t \\
\mathbf{y}_t - \Omega \mathbf{y}_{t-1} &= \mathbf{e}_t \\
(\mathbf{I}_2 - \Omega L)\, \mathbf{y}_t &= \mathbf{e}_t \\
\mathbf{y}_t &= (\mathbf{I}_2 - \Omega L)^{-1} \mathbf{e}_t \\
\mathbf{y}_t &= \left(\mathbf{I}_2 + \Omega L + \Omega^2 L^2 + \ldots\right) \mathbf{e}_t \\
\mathbf{y}_t &= \mathbf{e}_t + \Omega \mathbf{e}_{t-1} + \Omega^2 \mathbf{e}_{t-2} + \ldots
\end{aligned}
$$

- IRF are made of the elements pre-multiplying the different shocks (residuals)
- Represented as plots

Notes

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3.What is VAR(1) *Econometrics* **III**?

- Forecasting, e.g. next period

$$m_t, m_{t-1}, \ldots, x_t, x_{t-1}, \ldots \rightarrow m_{t+1}?$$
$$m_t, m_{t-1}, \ldots, x_t, x_{t-1}, \ldots \rightarrow x_{t+1}?$$

$$E[\mathbf{y}_{t+1}] = \hat{\mathbf{y}}_{t+1|t} = \Omega \mathbf{y}_t$$

- Forecasting horizon
- Forecasting accuracy → MSE

3.What is VAR(1) *Econometrics* **III**?

- MSE, e.g. next period and four periods ahead

$$\begin{aligned} \mathbf{MSE}(\hat{\mathbf{y}}_{t+1|t}) &= E[\mathbf{y}_{t+1} - \hat{\mathbf{y}}_{t+1|t}]^2 \\ &= E[\Omega \mathbf{y}_t + \mathbf{e}_{t+1} - \Omega \mathbf{y}_t]^2 \\ &= E[\mathbf{e}_{t+1}]^2 \\ &= Var[\mathbf{e}_{t+1}] \\ &= Var[\mathbf{e}] \end{aligned}$$

$$\begin{aligned} \mathbf{MSE}(\hat{\mathbf{y}}_{t+4|t}) &= E[\mathbf{y}_{t+4} - \hat{\mathbf{y}}_{t+4|t}]^2 \\ &= f(Var[\mathbf{e}_{t+1}], \ldots, Var[\mathbf{e}_{t+4}]) \\ &= f(Var[e^m], Var[e^x]) \end{aligned}$$

Notes

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3.What is VAR(1) *Econometrics* (**SVAR**)?1/3

- Presented VAR depend only on lagged variables
  - → Reduced form
- New form has a contemporaneous structure among variables
  - → SVAR

$$
\begin{aligned}
m_t &= \omega_{11} m_{t-1} + \omega_{12} x_{t-1} + e_t^m \\
x_t &= \omega_{21} m_{t-1} + \omega_{22} x_{t-1} + e_t^x
\end{aligned}
$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$
\begin{aligned}
b_{11}^0 m_t &= -b_{12}^0 x_t + b_{11}^1 m_{t-1} + b_{12}^1 x_{t-1} + \varepsilon_t^m \\
b_{22}^0 x_t &= -b_{21}^0 m_t + b_{21}^1 m_{t-1} + b_{22}^1 x_{t-1} + \varepsilon_t^x
\end{aligned}
$$

- Problem SVAR estimated as RF

3.What is VAR(1) *Econometrics* (**SVAR**)?2/3

- Matrix notation

$$
\begin{pmatrix} b_{11}^0 & b_{12}^0 \\ b_{21}^0 & b_{22}^0 \end{pmatrix}
\begin{pmatrix} m_t \\ x_t \end{pmatrix}
=
\begin{pmatrix} b_{11}^1 & b_{12}^1 \\ b_{21}^1 & b_{22}^1 \end{pmatrix}
\begin{pmatrix} m_{t-1} \\ x_{t-1} \end{pmatrix}
+
\begin{pmatrix} \varepsilon_t^m \\ \varepsilon_t^x \end{pmatrix}
$$

$$
\begin{aligned}
\mathbf{B}_0 \mathbf{y}_t &= \mathbf{B}_1 \mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t \\
\mathbf{y}_t &= \mathbf{B}_0^{-1} \mathbf{B}_1 \mathbf{y}_{t-1} + \mathbf{B}_0^{-1} \boldsymbol{\varepsilon}_t \\
\mathbf{y}_t &= \Omega \mathbf{y}_{t-1} + \mathbf{e}_t
\end{aligned}
$$

- Problem, more unknown parameters than information

$\Omega = 4$ reduced-form coefficients
$Var\,[\mathbf{e}] = 3$ pieces of information
$\rightarrow 7$ pieces of information

$\mathbf{B}_0 = 4$ structural coefficients
$\mathbf{B}_1 = 4$ structural coefficients
$Var\,[\boldsymbol{\varepsilon}] = 3$ pieces of information
$\rightarrow 11$ unknown structural coefficients

- 4 restrictions

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3.What is VAR(1) *Econometrics* (**SVAR**)?3/3

- Identification EViews 3.1

$$\begin{pmatrix} \boxed{1} & b_{12}^{0} \\ b_{21}^{0} & \boxed{1} \end{pmatrix} \begin{pmatrix} m_t \\ x_t \end{pmatrix} = \begin{pmatrix} b_{11}^{1} & b_{12}^{1} \\ b_{21}^{1} & b_{22}^{1} \end{pmatrix} \begin{pmatrix} m_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^m \\ \varepsilon_t^x \end{pmatrix}$$

$$\begin{pmatrix} Var\left[\varepsilon^m\right] & Cov \\ Cov & Var\left[\varepsilon^x\right] \end{pmatrix} = \begin{pmatrix} Var\left[\varepsilon^m\right] & \boxed{0} \\ \boxed{0} & Var\left[\varepsilon^x\right] \end{pmatrix}$$

$$\begin{pmatrix} 1 & \boxed{0} \\ b_{21}^{0} & 1 \end{pmatrix} \begin{pmatrix} m_t \\ x_t \end{pmatrix} = \begin{pmatrix} b_{11}^{1} & b_{12}^{1} \\ b_{21}^{1} & b_{22}^{1} \end{pmatrix} \begin{pmatrix} m_{t-1} \\ x_{t-1} \end{pmatrix} + \begin{pmatrix} \varepsilon_t^m \\ \varepsilon_t^x \end{pmatrix}$$

- So-called Cholesky identification or recursive identification

4.What have we learned with VAR?

- VAR versus large macro model
- Transmission mechanism of monetary policy (e.g. credit channel, money neutrality,…)
- Descriptive central bank reaction function ≅ rule
- Forecasting (e.g. expected inflation, …)
- Robustness of results

5.Just do it!

- EViews
  Estimate VAR → SVAR → Dynamics and Variance Decomposition → Analysis → Redo

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