Monopolistic competition, complement

Production function, quantity as a function of labor

$$q = \sqrt{L}$$

$$L = q^2$$

$$MPL = \frac{\partial q}{\partial L} = 0.5L^{-0.5} = \frac{1}{2q}$$

MC (derivative of cost function with respect to quantity), wage rate over MPL

$$MC = \frac{\omega}{\frac{1}{2q}} = 2\omega q$$

Profit $\Im$, turnover (output $\times$ price per unit of output) minus cost (wage rate $\times$ number of hours)

$$\Im = pq - \omega L$$

$$\Im = pq - \omega q^2$$

Optimization, profit maximization

$$\frac{\partial \Im}{\partial q} = 0$$

$$p'q + p \cdot 1 - 2\omega q = 0 \quad \rightarrow \quad p'q + p = 2\omega q$$

[Second derivative is $p' - 2\omega$ and should be negative]; Interpretation, this is $MR = MC$, MR has two effects, output (p, positive) and price effect ($p'q$, negative)

In the notes FOC (equation 12, page 7)

$$Y_t(i) + (P_t(i) - \psi_t)\frac{\partial Y_t(i)}{\partial P_t(i)} = 0$$

$$Y_t(i) + \frac{\partial Y_t(i)}{\partial P_t(i)} P_t(i) = \frac{\partial Y_t(i)}{\partial P_t(i)} \psi_t$$

$$\frac{Y_t(i)}{\frac{\partial Y_t(i)}{\partial P_t(i)}} + P_t(i) = \psi_t \quad \rightarrow \quad \frac{\partial P_t(i)}{\partial Y_t(i)} Y_t(i) + P_t(i) = \psi_t$$

This is $MR = MC$, price effect and output effect for the MR equals the marginal cost

Demand function, complement

Price elasticity of demand $Y_t(i) = \left(\frac{P_t}{P_t(i)}\right)^{\frac{1}{1-q}} Y_t$

$$\ln(Y_t(i)) = \frac{1}{1-q}\left(\ln(P_t) - \ln(P_t(i))\right) + \ln(Y_t)$$

$$\frac{d\ln(Y_t(i))}{d\ln(P_t(i))} = -\frac{1}{1-q}$$

Elasticity of substitution, complement

$$Y_t = \left[\int Y_t\,(i)^q \,\mathtt{d}i\right]^{\frac{1}{q}} \qquad 0 < q < 1$$

$$Y_t = [\cdots + Y_t(ii)^q + Y_t(iii)^q + \dots]^{\frac{1}{q}}$$

$$dY_t = \left[\cdots + dY_t(ii)Y_t(ii)^{q-1} + dY_t(iii)Y_t(iii)^{q-1} + \dots\right]^{\frac{1}{q}-1}$$

$$0 = \left[dY_t(ii)Y_t(ii)^{q-1} + dY_t(iii)Y_t(iii)^{q-1}\right]^{\frac{1-q}{q}}$$

$$\frac{dY_t(ii)}{dY_t(iii)} = -\left(\frac{Y_t(ii)}{Y_t(iii)}\right)^{1-q}$$

$$\frac{d\ln\frac{Y_t(ii)}{Y_t(iii)}}{d\ln|MRS|} = \left(\frac{d\ln|MRS|}{d\ln\frac{Y_t(ii)}{Y_t(iii)}}\right)^{-1} = \frac{1}{1-q}$$

q near zero means high markup (a lot of pricing power), low price elasticity, low elasticity of substitution

q near one means low markup (almost no pricing power), high price elasticity, high elasticity of substitution