Economics of central banking (Zentralbanktheorie und Zentralbankpolitik)

Supplementary slides

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These slides present the main elements of the course 7,280; they are based on several papers and books (e.g. Walsh, 2003, Galí, 2008) whose references are given during the term; a separate reading list for the exam will be given after the term break.

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Solution Example Question 8, Equilibrium inflation rate

Utility of conservative central banker, with supply curve y

$$U = -(y-k)^2 - (1+\delta)\pi^2$$
$$U = -(\pi - \pi^e + \varepsilon - k)^2 - (1+\delta)\pi^2$$

group elements in the first parenthesis

$$U = -((\pi - \pi^e) + (\varepsilon - k))^2 - (1+\delta)\pi^2$$

use formula $(a+b)^2 = a^2 + b^2 + 2ab$

$$U = -\Big((\pi - \pi^e)^2 + (\varepsilon - k)^2 + 2(\varepsilon - k)(\pi - \pi^e)\Big) - (1+\delta)\pi^2$$

use formula again for the other square elements

$$U = -\pi^2 - (\pi^e)^2 + 2\pi\pi^e - \varepsilon^2 - k^2 + 2k\varepsilon - 2\varepsilon\pi + 2\varepsilon\pi^e + 2k\pi - 2k\pi^e - (1+\delta)\pi^2$$

take expectation, the central banker observes ε; $\mathbf{E}\varepsilon^2$ is the variance of the output shock

$$\mathbf{E}U = -\pi^2 - (\pi^e)^2 + 2\pi\pi^e - \sigma_\varepsilon^2 - k^2 + 2k\varepsilon - 2\varepsilon\pi + 2\varepsilon\pi^e + 2k\pi - 2k\pi^e - (1+\delta)\pi^2$$

maximize the utility, take first derivative wrt π, and set it equal to zero

$$0 = -2\pi + 2\pi^e - 2\varepsilon + 2k - 2(1+\delta)\pi$$

solve for the inflation rate as a function of the expected inflation rate by the economic agents

$$\pi = \frac{\pi^e - \varepsilon + k}{2+\delta}$$

compute the inflation expectations of agents, which do not observe ε (its mean equals zero)

$$\mathbf{E}\pi = \frac{\pi^e - \mathbf{E}\varepsilon + k}{2+\delta} = \pi^e$$

solve for π^e

$$\pi^e = \frac{k}{1+\delta}$$

plug in $\pi = \frac{\pi^e - \varepsilon + k}{2+\delta}$

$$\pi = \frac{\frac{k}{1+\delta} - \varepsilon + k}{2+\delta}$$

and get the equilibrium inflation rate

$$\pi = \frac{k}{1+\delta} - \frac{\varepsilon}{2+\delta}$$

Solution Example Question 8, Optimal degree of conservatism

Plug π^e and π in the utility function of the society $U = -(\pi - \pi^e + \varepsilon - k)^2 - \pi^2$ (after having used the definition of the supply curve)

$$U = -\left(\frac{k}{1+\delta} - \frac{\varepsilon}{2+\delta} - \frac{k}{1+\delta} + \varepsilon - k\right)^2 - \left(\frac{k}{1+\delta} - \frac{\varepsilon}{2+\delta}\right)^2$$

rewrite

$$U = -\left(\frac{1+\delta}{2+\delta}\varepsilon - k\right)^2 - \left(\frac{k}{1+\delta} - \frac{\varepsilon}{2+\delta}\right)^2$$

take expectation, knowing that the element with ε are zero and the elements with ε^2 become σ_ε^2

$$\mathbf{E}U = -\left(\left(\frac{1+\delta}{2+\delta}\right)^2 \sigma_\varepsilon^2 + k^2\right) - \left(\left(\frac{k}{1+\delta}\right)^2 + \frac{\sigma_\varepsilon^2}{(2+\delta)^2}\right)$$

group similar elements (i.e. σ_ε^2 and k^2)

$$\mathbf{E}U = -\left[\left[(1+\delta)^2 + 1\right]\left(\frac{\sigma_\varepsilon^2}{(2+\delta)^2} + \frac{k^2}{(1+\delta)^2}\right)\right]$$

maximize the utility, take first derivative wrt δ, and set it equal to zero

$$-\left[2(1+\delta)\left(\frac{\sigma_\varepsilon^2}{(2+\delta)^2} + \frac{k^2}{(1+\delta)^2}\right) + \left[(1+\delta)^2 + 1\right]\left(-\frac{2\sigma_\varepsilon^2}{(2+\delta)^3} - \frac{2k^2}{(1+\delta)^3}\right)\right] = 0$$

rewrite

$$-(1+\delta)\left(\frac{\sigma_\varepsilon^2}{(2+\delta)^2} + \frac{k^2}{(1+\delta)^2}\right) + \left[(1+\delta)^2 + 1\right]\left(\frac{\sigma_\varepsilon^2}{(2+\delta)^3} + \frac{k^2}{(1+\delta)^3}\right) = 0$$

rewrite

$$\frac{\left[(1+\delta)^2+1\right]\sigma_\varepsilon^2}{(2+\delta)^3}-\frac{(1+\delta)\,\sigma_\varepsilon^2}{(2+\delta)^2}+\frac{\left[(1+\delta)^2+1\right]k^2}{(1+\delta)^3}-\frac{(1+\delta)\,k^2}{(1+\delta)^2}=0$$

which gives

$$\delta\frac{\sigma_\varepsilon^2}{(2+\delta)^3}=\frac{k^2}{(1+\delta)^3}$$

or simplified

$$\delta=\frac{k^2}{\sigma_\varepsilon^2}\frac{(2+\delta)^3}{(1+\delta)^3}$$

is the value $\delta=0$ a solution?

$$0=\frac{8k^2}{\sigma_\varepsilon^2}$$

no, it means that it is worth hiring a conservative central banker in this setup; and if there is no inflation bias ($k=0$)? then $\delta=0$ is a solution, meaning that it is not worth hiring a conservative central banker; this makes sense because in the first place a conservative central banker would be appointed to get rid of the inflation bias

Solution Example Question 9

- Price elasticity of the following demand function

$$Y_t(i) = \left(\frac{P_t}{P_t(i)}\right)^{\frac{1}{1-q}} Y_t$$

  – solution, quick way, using log

$$\ln Y_t(i) = \frac{1}{1-q}\ln P_t - \frac{1}{1-q}\ln P_t(i) + \ln Y_t$$

$$\frac{\partial \ln Y_t(i)}{\partial \ln P_t(i)} = -\frac{1}{1-q}$$

  – solution, long way, using $\frac{\partial Y_t(i)}{\partial P_t(i)}\frac{P_t(i)}{Y_t(i)}$, compute first the derivative

$$\frac{\partial Y_t(i)}{\partial P_t(i)} = \frac{1}{1-q}\left(\frac{P_t}{P_t(i)}\right)^{\frac{1}{1-q}-1} Y_t \frac{-P_t}{P_t(i)\,P_t(i)}$$

  which simplifies

$$\frac{\partial Y_t(i)}{\partial P_t(i)} = -\frac{1}{1-q}\frac{Y_t(i)}{P_t(i)}$$

  together with the definition, it gives the elasticity

- Elasticity of substitution of production function Y_t

$$Y_t = \left[\int Y_t(i)^q \,\mathrm{d}i\right]^{\frac{1}{q}}$$

- rewrite

$$Y_t = \left[\ldots + Y_t(i_1)^q + Y_t(i_2)^q + \ldots\right]^{\frac{1}{q}}$$

- total differential

$$\begin{aligned} \mathtt{d}Y_t &= \left[\ldots + Y_t(i_1)^q + Y_t(i_2)^q + \ldots\right]^{\frac{1}{q}-1} \\ &\quad \times \left[\ldots + Y_t(i_1)^{q-1}\,\mathtt{d}Y_t(i_1) + Y_t(i_2)^{q-1}\,\mathtt{d}Y_t(i_2) + \ldots\right] \end{aligned}$$

- restrictions

$$\mathtt{d}Y_t = \mathtt{d}Y_t(i_3) = \mathtt{d}Y_t(i_4) = \ldots = 0$$

- apply to $\mathtt{d}Y_t$ to get the TRS

$$0 = Y_t(i_1)^{q-1}\,\mathtt{d}Y_t(i_1) + Y_t(i_2)^{q-1}\,\mathtt{d}Y_t(i_2)$$

$$\frac{\mathtt{d}Y_t(i_1)}{\mathtt{d}Y_t(i_2)} = -\left(\frac{Y_t(i_2)}{Y_t(i_1)}\right)^{q-1}$$

- compute $\ln|TRS|$

$$(q-1)\ln\frac{Y_t(i_2)}{Y_t(i_1)}$$

$$(1-q)\ln\frac{Y_t(i_1)}{Y_t(i_2)}$$

- compute $\frac{\mathtt{d}\ln|TRS_t|}{\mathtt{d}\ln\frac{Y_t(i_1)}{Y_t(i_2)}}$

$$1-q$$

– elasticity of substitution

$$\frac{\mathtt{d}\ln\frac{Y_t(i_1)}{Y_t(i_2)}}{\mathtt{d}\ln|TRS_t|}=\frac{1}{1-q}$$

Solution Example Question 11

- Random walk, AR(1), $x_t = \alpha + \beta x_{t-1} + \varepsilon_t$, with unit root, $\beta = 1$

$$\begin{aligned} x_t &= \alpha + x_{t-1} + \varepsilon_t \\ \mathbf{E} x_{t+1} &= \alpha + x_t + \mathbf{E}\varepsilon_{t+1} = \alpha + x_t \end{aligned}$$

- Growth rates, see Excel sheet in class