

Economics of central banking (Zentralbanktheorie und Zentralbankpolitik)

Matrix algebra, identification

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These slides present the main elements of the course 7,280; they are based on several papers and books (e.g. Walsh, 2003, Galí, 2008) whose references are given during the term; a separate reading list for the exam will be given after the term break.

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Pro memoria: matrix algebra

- 'Reduction' of a system

$$x_1 = 3a + 4b$$

$$x_2 = 5b$$

becomes

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 & 4 \\ 0 & 5 \end{bmatrix} \times \begin{bmatrix} a \\ b \end{bmatrix}$$

or

$$\mathbf{x} = \mathbf{A}\mathbf{v}$$

- Multiplication, 1st example

$$\begin{bmatrix} a & b \end{bmatrix} \times \begin{bmatrix} c \\ d \end{bmatrix} = ac + bd$$

- Multiplication, 2nd example

$$\begin{bmatrix} c \\ d \end{bmatrix} \times \begin{bmatrix} a & b \end{bmatrix} = \begin{bmatrix} ca & cb \\ da & db \end{bmatrix}$$

- Multiplication, 3rd example

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} ae + bg & af + bh \\ ce + dg & cf + dh \end{bmatrix}$$

- Premultiplication, postmultiplication, multiplication is not commutative

$$\mathbf{AB} \neq \mathbf{BA}$$

□ Size matters, conformable matrices

□ Inverse

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = \mathbf{I} = \begin{bmatrix} 1 & 0 & \dots \\ 0 & 1 & \\ \vdots & & \ddots \end{bmatrix}$$

□ Sum

$$\begin{bmatrix} a & b \end{bmatrix} + \begin{bmatrix} c & d \end{bmatrix} = \begin{bmatrix} a + c & b + d \end{bmatrix}$$

□ Literature

- books on mathematics, econometrics, or statistics
- eg. Econometric Analysis, William Greene, 6th edition, Prentice Hall, Appendix A

Identification

- (just)identified

$$7 = 9 - X$$

$$\rightarrow X = 2$$

- underidentified

$$7 = X + Y$$

\rightarrow assumption $Y = 1$, which implies $X = 6$

- overidentified

$$10 = 12 - X$$

$$45 = 40 + X$$

\rightarrow needs to optimize